NASA Technical Memorandum 85912

Combining and Connecting Linear, Multi-Input, Multi-Output Subsystem Models

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SUMMARY

This report provides the mathematical background for combining and connecting linear, multi-input, multi-output subsystem models into an overall systems model. Several examples of subsystem configurations are examined in detail. The report also contains a description of a $MATRIX_x$ command file to aid in the process of combining and connecting these subsystem models.

INTRODUCTION

To analyze and design models for complex systems, it is essential to combine subsystem models into an overall system model. This report provides the mathematical background for combining and connecting linear, multi-input, multi-output subsystem models. Throughout this report, systems are assumed to have a standard form.

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} + D\underline{u}$$

Using this system model, the mathematics that combine and connect two systems at a time are described. Six representative combinations are presented: parallel systems (fig. 1), summation of systems (fig. 2), concatenation of systems (fig. 3), systems with a common input (fig. 4), summation of systems with a common input (fig. 5), and feedback systems (fig. 6). The matrices representing the combined system are presented in terms of the matrices for the subsystems.

The mathematics used in the development of the sample systems are presented in appendix A in a $MATRIX_x$ (ref. 1) command file format. A listing of the command file and instructions for the use of the file are presented in this report. $MATRIX_x$ is a general purpose, interactive program for matrix manipulation, control law analysis and design, and parameter identification.*

NOMENCLATURE

n dimension of state vector, $\underline{x}$

m dimension of control vector, $\underline{u}$

r dimension of observation vector, $\underline{y}$

s Laplace variable

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*$MATRIX_x$ is a proprietary product of Integrated Systems Inc., Palo Alto, California.

Matrices:

A	state matrix
B	control matrix
C	observation matrix
D	feedforward matrix
F	input selection matrix
H	output selection matrix
I	identity matrix
N	$(I + D_1 H_2 D_2 F_2)^{-1}$
0	matrix of zeros

Vectors:

$\underline{u}$	control vector
$\underline{w}$	selected input vector
$\underline{x}$	state vector
$\underline{y}$	observation vector
$\underline{z}$	selected output vector

Subscripts:

1	system one
2	system two
$\underline{1}$	upper left submatrix of a partitioned matrix
$\underline{2}$	upper right submatrix of a partitioned matrix
$\underline{3}$	lower left submatrix of a partitioned matrix
$\underline{4}$	lower right submatrix of a partitioned matrix

MATHEMATICAL PRELIMINARIES

Matrices, which are a convenient method of formulating simultaneous linear equations, are extremely useful in representing the linear models of dynamic systems. Just as a system of linear scalar equations can be transformed into a matrix equation, a system of linear matrix equations can be transformed into a matrix equation using partitioned matrices. Thus the process for converting the scalar equations

$$\begin{aligned} a_{1_1} x_1 + \cdots + a_{1_n} x_n &= b_1 \\ &\vdots \\ a_{n_1} x_1 + \cdots + a_{n_n} x_n &= b_n \end{aligned}$$

into the matrix equation

$$\begin{bmatrix} a_{1_1} & \cdots & a_{1_n} \\ & \vdots & \\ a_{n_1} & \cdots & a_{n_n} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

is identical to that used for converting the matrix equations

$$\begin{aligned} A_{1_1} \underline{x}_1 + \cdots + A_{1_n} \underline{x}_n &= \underline{b}_1 \\ &\vdots \\ A_{n_1} \underline{x}_1 + \cdots + A_{n_n} \underline{x}_n &= \underline{b}_n \end{aligned}$$

into the partitioned matrix equation

$$\left[\begin{array}{c|c|c} A_{1_1} & \cdots & A_{1_n} \\ \hline & \vdots & \\ \hline A_{n_1} & \cdots & A_{n_n} \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline \vdots \\ \hline \underline{x}_n \end{array}\right] = \left[\begin{array}{c} \underline{b}_1 \\ \hline \vdots \\ \hline \underline{b}_n \end{array}\right]$$

Partitioned matrices are manipulated in the same way as normal matrices except that the matrices and partitioned submatrices must be conformable for the operation being performed. Consider the addition of matrices A and B, each partitioned into four submatrices.

$$A = \left[\begin{array}{c|c} \underline{A_1} & \underline{A_2} \\ \hline \underline{A_3} & \underline{A_4} \end{array}\right]$$

$$B = \left[\begin{array}{c|c} B_1 & B_2 \\ \hline B_3 & B_4 \end{array}\right]$$

If A and B have the same order and the corresponding submatrices (for example, A_1 and B_1) are of the same order, the sum may be expressed in terms of the partitioned submatrices.

$$A + B = \left[\begin{array}{c|c} A_1 + B_1 & A_2 + B_2 \\ \hline A_3 + B_3 & A_4 + B_4 \end{array}\right]$$

The same principles hold for multiplication. For example, if matrices A and B are conformable for multiplication and all appropriate partitioned submatrices are conformable for multiplication, then the product can be represented as

$$AB = \left[\begin{array}{c|c} A_1B_1 + A_2B_3 & A_1B_2 + A_2B_4 \\ \hline A_3B_4 + A_4B_3 & A_3B_2 + A_4B_4 \end{array}\right]$$

DEVELOPMENT OF COMBINED SYSTEM MODELS

In this section, the models for six representative subsystem combinations are developed. These subsystem combinations are for two independent (parallel) systems (fig. 1), the summation of two systems (fig. 2), the concatenation of two systems (fig. 3), two systems with a common input (fig. 4), the summation of two systems with a common input (fig. 5), and a feedback system (fig. 6). Each subsystem considered is assumed to be represented by linear system equations of the form

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} + D\underline{u}$$

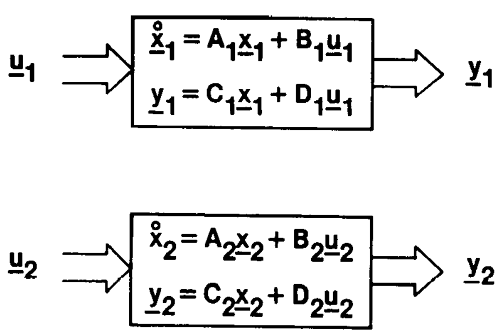


Figure 1. Parallel systems.

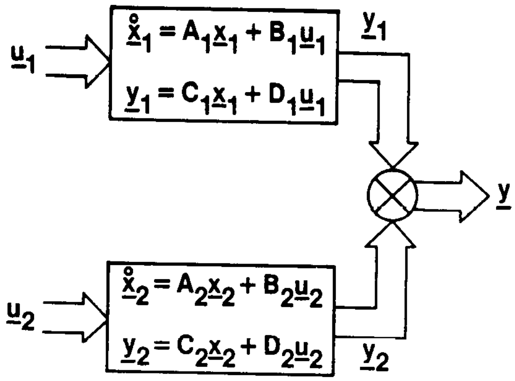


Figure 2. Summation of systems.

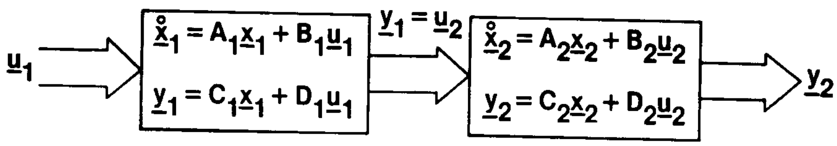


Figure 3. Concatenation of systems.

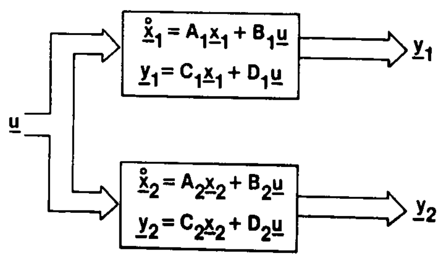


Figure 4. Systems with a common input.

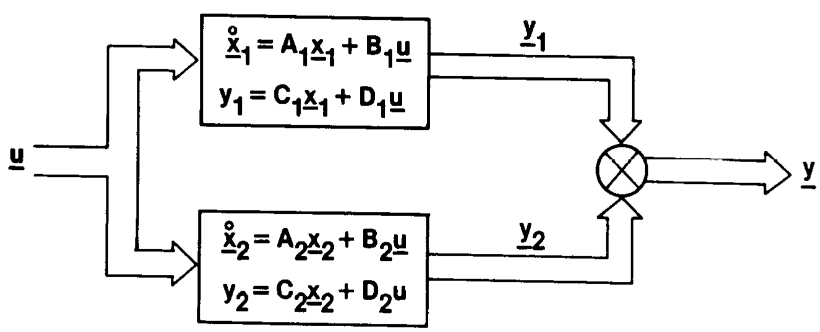


Figure 5. Summation of systems with a common input.

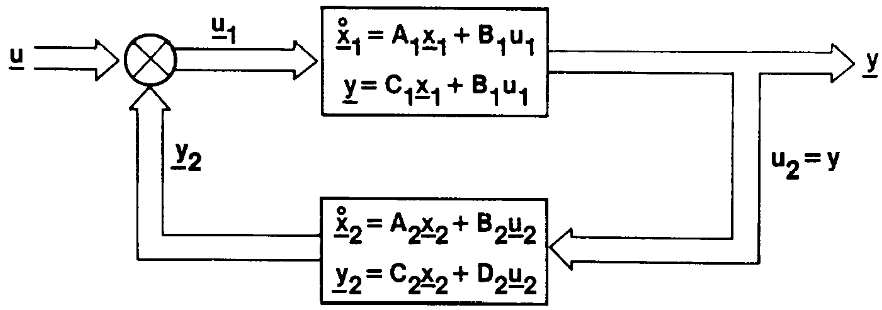


Figure 6. Feedback system.

The state vector $\underline{x}$ is assumed to be an $n \times 1$ vector. The control vector $\underline{u}$ and the observation vector $\underline{y}$ are assumed to be $m \times 1$ and $r \times 1$, respectively.

For the systems in which subsystems are connected, the concept of a selection matrix is included. For notational convenience, the concept of an input and an output selection matrix is used. These matrices are denoted F and H, respectively, and are related to the subsystem control and observation vectors by the following definitions.

$$\underline{u} = F\underline{w}$$

$$\underline{z} = H\underline{y}$$

The vector $\underline{w}$ represents a generalized input into the system being considered. The vector $\underline{z}$ serves the same purpose for output.

For example, an output selection matrix that is not the identity matrix could be used as a feedback controller model that is developed independently of the plant model. If the feedback control vector, $\underline{u}$, of the controller model were

$$\underline{u} = \begin{bmatrix} q \\ \alpha \\ \theta \end{bmatrix}$$

and the observation vector $\underline{y}$ of the plant model were

$$\underline{y} = \begin{bmatrix} a_n \\ q \\ \theta \\ \alpha \end{bmatrix}$$

the selection matrix would be used to select the last three elements of the plant observation vector and reorder them to conform to the order of the feedback control vector for the controller. For this example, the selection matrix H would be

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

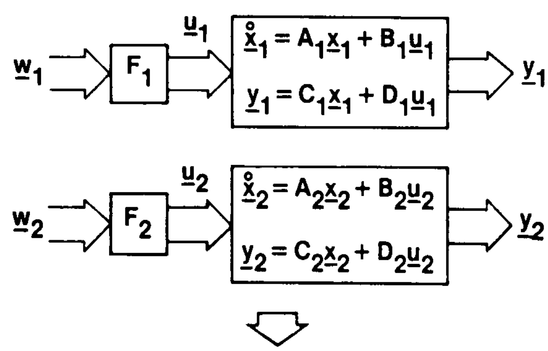
If scaling of output parameters is desired, the elements of the selection matrix can be used to accommodate that scaling. The use of the input selection matrix F is identical to that described previously for the output selection matrix.

Parallel Systems

Two independent subsystems having no interconnections can be directly combined into a single system model simply by defining the new control, state, and observation vectors and then, by inspection, constructing the partitioned matrices of the resultant combined system. Figure 7 illustrates two parallel systems governed by the following equations.

$$\left.\begin{aligned} \dot{\underline{x}}_1 &= A_1\underline{x}_1 + B_1\underline{u}_1 \\ \underline{y}_1 &= C_1\underline{x}_1 + D_1\underline{u}_1 \end{aligned}\right\} \text{System 1}$$

$$\left.\begin{aligned} \dot{\underline{x}}_2 &= A_2\underline{x}_2 + B_2\underline{u}_2 \\ \underline{y}_2 &= C_2\underline{x}_2 + D_2\underline{u}_2 \end{aligned}\right\} \text{System 2}$$



$$\left[\begin{array}{c} \dot{\underline{x}}_1 \\ \hline \dot{\underline{x}}_2 \end{array}\right] = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_2 \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline \underline{x}_2 \end{array}\right] + \left[\begin{array}{c|c} B_1F_1 & 0 \\ \hline 0 & B_2F_2 \end{array}\right] \left[\begin{array}{c} \underline{w}_1 \\ \hline \underline{w}_2 \end{array}\right]$$

$$\left[\begin{array}{c} \underline{y}_1 \\ \hline \underline{y}_2 \end{array}\right] = \left[\begin{array}{c|c} C_1 & 0 \\ \hline 0 & C_2 \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline \underline{x}_2 \end{array}\right] + \left[\begin{array}{c|c} D_1F_1 & 0 \\ \hline 0 & D_2F_2 \end{array}\right] \left[\begin{array}{c} \underline{w}_1 \\ \hline \underline{w}_2 \end{array}\right]$$

Figure 7. State representation of combined parallel systems.

The control vectors to these systems are defined in terms of the input vectors $\underline{w}_1$ and $\underline{w}_2$, using the input selection matrices $\underline{F}_1$ and F_2.

$$\underline{u}_1 = F_1\underline{w}_1 \quad \text{System 1}$$

$$\underline{u}_2 = F_2\underline{w}_2 \quad \text{System 2}$$

The control, state, and observation vectors are defined to include all elements contained in the subsystem models.

$$\underline{u} = \begin{bmatrix} \underline{w}_1 \\ \hline \underline{w}_2 \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} \underline{y}_1 \\ \hline \underline{y}_2 \end{bmatrix}$$

If the system state equations are expressed in terms of the input vectors and the input selection matrices, the equations for the system become

$$\dot{\underline{x}}_1 = A_1x_1 + B_1F_1\underline{w}_1 \quad \text{System 1}$$

$$\dot{\underline{x}}_2 = A_2\underline{x}_2 + B_2F_2\underline{w}_2 \quad \text{System 2}$$

Using the previous definitions, the new state equation is represented by the following equations.

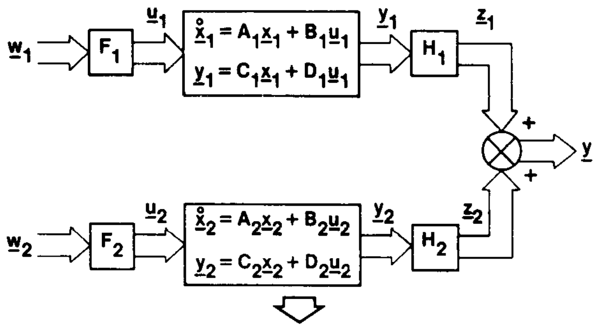
$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \hline \dot{\underline{x}}_2 \end{bmatrix} = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix} + \left[\begin{array}{c|c} B_1F_1 & 0 \\ \hline 0 & B_2F_2 \end{array}\right] \begin{bmatrix} \underline{w}_1 \\ \hline \underline{w}_2 \end{bmatrix}$$

$$\begin{bmatrix} \underline{y}_1 \\ \hline \underline{y}_2 \end{bmatrix} = \left[\begin{array}{c|c} C_1 & 0 \\ \hline 0 & C_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix} + \left[\begin{array}{c|c} D_1F_1 & 0 \\ \hline 0 & D_2F_2 \end{array}\right] \begin{bmatrix} \underline{w}_1 \\ \hline \underline{w}_2 \end{bmatrix}$$

The zero submatrices are of an order appropriate for the application.

Summation of Systems

Figure 8 illustrates the summation of two systems having independent input vectors. As shown in the figure, the systems are preceded by an input selection matrix and followed by an output selection matrix. The combined system model is developed by defining the control, state, and output vectors for the total system; expressing the subsystem controls in terms of the input vectors and input selection matrices; and then expressing the output vector of the total system in terms of the selection and observation equation matrices for both subsystems.



$$\begin{bmatrix} \mathring{\underline{x}}_1 \\ \mathring{\underline{x}}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} B_1F_1 & 0 \\ 0 & B_2F_2 \end{bmatrix} \begin{bmatrix} \underline{w}_1 \\ \underline{w}_2 \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} H_1C_1 & H_2C_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} H_1D_1F_1 & H_2D_2F_2 \end{bmatrix} \begin{bmatrix} \underline{w}_1 \\ \underline{w}_2 \end{bmatrix}$$

Figure 8. State representation for summation of systems.

The system equations for the two subsystems are given as

$$\left.\begin{aligned} \dot{\underline{x}}_1 &= A_1\underline{x}_1 + B_1\underline{u}_1 \\ \underline{y}_1 &= C_1\underline{x}_1 + D_1\underline{u}_1 \end{aligned}\right\} \text{System 1}$$

$$\left.\begin{aligned} \dot{\underline{x}}_2 &= A_2\underline{x}_2 + B_2\underline{u}_2 \\ \underline{y}_2 &= C_2\underline{x}_2 + D_2\underline{u}_2 \end{aligned}\right\} \text{System 2}$$

The control vectors for the two systems are defined in forms of input vectors and input selection matrices.

$$\underline{u}_1 = F_1\underline{w}_1 \quad \text{System 1}$$

$$\underline{u}_2 = F_2\underline{w}_2 \quad \text{System 2}$$

Substituting these expressions into the state equations for both systems yields equations in terms of the system inputs, the system selection matrices, the original state vector, and the state and control matrices.

$$\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1F_1\underline{w}_1 \quad \text{System 1}$$
$$\dot{\underline{x}}_2 = A_2\underline{x}_2 + B_2F_2\underline{w}_2 \quad \text{System 2}$$

The output of the total system, $\underline{y}$, is the sum of the selected outputs of system 1 and system 2, $\underline{z}_1$ and $\underline{z}_2$, respectively.

$$\underline{y} = \underline{z}_1 + \underline{z}_2$$

The selected vectors are defined as

$$\left.\begin{aligned} z_1 &= H_1\underline{y}_1 \\ &= H_1C_1\underline{x}_1 + H_1D_1F_1\underline{w}_1 \end{aligned}\right\} \text{System 1}$$

$$\left.\begin{aligned} \underline{z}_2 &= H_2\underline{y}_2 \\ &= H_2C_2\underline{x}_2 + H_2D_2F_2\underline{w}_2 \end{aligned}\right\} \text{System 2}$$

Thus the output vector for the total system can be written

$$\underline{y} = H_1C_1\underline{x}_1 + H_2C_2\underline{x}_2 + H_1D_1F_1\underline{w}_1 + H_2D_2F_2\underline{w}_2$$

The control vector $\underline{u}$ and state vector $\underline{x}$ in the total system are defined as all elements that are to be added in the subsystems. This definition yields the following partitioned vectors.

$$\underline{u} = \left[\begin{array}{c} \underline{w}_1 \\ \hline \underline{w}_2 \end{array}\right]$$

$$\underline{x} = \left[\begin{array}{c} \underline{x}_1 \\ \hline \underline{x}_2 \end{array}\right]$$

The state equation can be written in terms of these vectors by inspection.

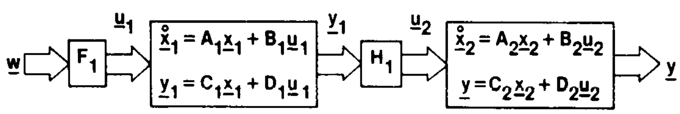
$$\left[\begin{array}{c} \dot{\underline{x}}_1 \\ \hline \dot{\underline{x}}_2 \end{array}\right] = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_2 \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline \underline{x}_2 \end{array}\right] + \left[\begin{array}{c|c} B_1F_1 & 0 \\ \hline 0 & B_2F_2 \end{array}\right] \left[\begin{array}{c} \underline{w}_1 \\ \hline \underline{w}_2 \end{array}\right]$$

The observation equation for the total system can be written using the previous definitions for the control and state vectors for the total system and the expanded observation equation.

$$\underline{y} = \left[\begin{array}{c|c} H_1C_1 & H_2C_2 \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline \underline{x}_2 \end{array}\right] + \left[\begin{array}{c|c} H_1D_1F_1 & H_2D_2F_2 \end{array}\right] \left[\begin{array}{c} \underline{w}_1 \\ \hline \underline{w}_2 \end{array}\right]$$

Concatenation of Systems

Two systems concatenated in a series are illustrated in figure 9. The first system contains an input selection and output selection matrix. The total system representation is determined by defining the state, control, and observation vectors for the total system, defining the output selection equation, and defining the observation equation from the first system to establish new system equations for the second system.



$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} = \left[\begin{array}{c|c} A_1 & 0 \\ \hline B_2H_1C_1 & A_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} B_1F_1 \\ \hline B_2H_1D_1F_1 \end{bmatrix} \underline{w}$$

$$\underline{y} = \left[\begin{array}{c|c} D_2H_1C_1 & C_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \left[D_2H_1D_1F_1\right] \underline{w}$$

Figure 9. State representation for concatenated systems.

The equations defining the two systems are

$$\left.\begin{aligned} \dot{\underline{x}}_1 &= A_1\underline{x}_1 + B_1\underline{u}_1 \\ \underline{y}_1 &= C_1\underline{x}_1 + D_1\underline{u}_1 \end{aligned}\right\} \text{System 1}$$

$$\left.\begin{aligned} \dot{\underline{x}}_2 &= A_2\underline{x}_2 + B_2\underline{u}_2 \\ \underline{y}_2 &= C_2\underline{x}_2 + D_2\underline{u}_2 \end{aligned}\right\} \text{System 2}$$

The control vector to the first system is determined by the input to the system and the input selection matrix.

$$\underline{u}_1 = F_1\underline{w}$$

The equations for the first system can be rewritten as

$$\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1F_1\underline{w}$$

$$\underline{y}_1 = C_1\underline{x}_1 + D_1F_1\underline{w}$$

The control vector for the second system is defined by the output selection matrix and the observation vector from the first system.

$$\underline{u}_2 = H_1\underline{y}_1$$

$$= H_1C_1\underline{x}_1 + H_1D_1F_1\underline{w}$$

The state vector $\underline{x}$, the control vector $\underline{u}$, and the observation vector $\underline{y}$ for the total system are defined as

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix}$$

$$\underline{u} = \underline{w}$$

$$\underline{y} = \underline{y}_2$$

The system equations for the second system must be reformulated in terms of $\underline{x}_1$, $\underline{x}_2$, $\underline{w}$, $\underline{y}_2$, and the system and selection matrices. Using the expanded definition of the control vector for the second system, the reformulated equations become

$$\dot{\underline{x}}_2 = A_2\underline{x}_2 + B_2H_1C_1\underline{x}_1 + B_2H_1D_1F_1\underline{w}$$

$$\underline{y}_2 = C_2\underline{x}_2 + D_2H_1C_1\underline{x}_1 + D_2H_1D_1F_1\underline{w}$$

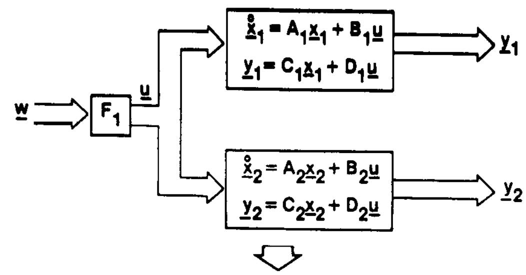
Using these equations and the expanded equations for the first system (in terms of $\underline{w}$), the state equations for the total system can be formulated.

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \hline \dot{\underline{x}}_2 \end{bmatrix} = \left[\begin{array}{c|c} A_1 & 0 \\ \hline B_2H_1C_1 & A_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix} + \begin{bmatrix} B_1F_1 \\ \hline B_2H_1D_1F_1 \end{bmatrix} \underline{w}$$

$$\underline{y} = \left[\begin{array}{c|c} D_2H_1C_1 & C_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix} + \left[D_2H_1D_1F_1\right] \underline{w}$$

Systems With a Common Input

Figure 10 illustrates two systems with a common input preceded by an input selection matrix. The formulation of the partitioned matrices for the total system is straightforward once the state equations are rewritten in terms of the input vector and input selection matrix.



$$\begin{bmatrix} \mathring{\underline{x}}_1 \\ \hline \mathring{\underline{x}}_2 \end{bmatrix} = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix} + \begin{bmatrix} B_1F_1 \\ \hline B_2F_1 \end{bmatrix} \underline{w}$$

$$\begin{bmatrix} \underline{y}_1 \\ \hline \underline{y}_2 \end{bmatrix} = \left[\begin{array}{c|c} C_1 & 0 \\ \hline 0 & C_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix} + \begin{bmatrix} D_1F_1 \\ \hline D_2F_1 \end{bmatrix} \underline{w}$$

Figure 10. State representation of systems with a common input.

The equations for the two systems are.

$$\left.\begin{aligned} \dot{\underline{x}}_1 &= A_1\underline{x}_1 + B_1\underline{u} \\ \underline{y}_1 &= C_1\underline{x}_1 + D_1\underline{u} \end{aligned}\right\} \text{System 1}$$

$$\left.\begin{aligned} \dot{\underline{x}}_2 &= A_2\underline{x}_2 + B_2\underline{u} \\ \underline{y}_2 &= C_2\underline{x}_2 + D_2\underline{u} \end{aligned}\right\} \text{System 2}$$

The control vector ($\underline{u}$) for the two systems is defined in terms of the input vector $\underline{w}$ and the input selection matrix F_1.

$$\underline{u} = F_1\underline{w}$$

The state vector $\underline{x}$, the control vector $\underline{u}$, and the observation vector $\underline{y}$ for the total system are defined as follows.

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix}$$

$$\underline{u} = \underline{w}$$

$$\underline{y} = \begin{bmatrix} \underline{y}_1 \\ \hline \underline{y}_2 \end{bmatrix}$$

Using the definition of the control vector common to both systems, the system equations can be rewritten.

$$\left.\begin{aligned}\dot{\underline{x}}_1 &= A_1\underline{x}_1 + B_1F_1\underline{w}\\ \underline{y}_1 &= C_1\underline{x}_1 + D_1F_1\underline{w}\end{aligned}\right\}\ \text{System 1}$$

$$\left.\begin{aligned}\dot{\underline{x}}_2 &= A_2\underline{x}_2 + B_2F_1\underline{w}\\ \underline{y}_2 &= C_2\underline{x}_2 + D_2F_1\underline{w}\end{aligned}\right\}\ \text{System 2}$$

Thus, by inspection, the total system can be formulated as

$$\begin{bmatrix}\dot{\underline{x}}_1\\ \hline \dot{\underline{x}}_2\end{bmatrix} = \left[\begin{array}{c|c}A_1 & 0\\ \hline 0 & A_2\end{array}\right]\begin{bmatrix}\underline{x}_1\\ \hline \underline{x}_2\end{bmatrix} + \begin{bmatrix}B_1F_1\\ \hline B_2F_1\end{bmatrix}\underline{w}$$

$$\begin{bmatrix}\underline{y}_1\\ \hline \underline{y}_2\end{bmatrix} = \left[\begin{array}{c|c}C_1 & 0\\ \hline 0 & C_2\end{array}\right]\begin{bmatrix}\underline{x}_1\\ \hline \underline{x}_2\end{bmatrix} + \begin{bmatrix}D_1F_1\\ \hline D_2F_1\end{bmatrix}\underline{w}$$

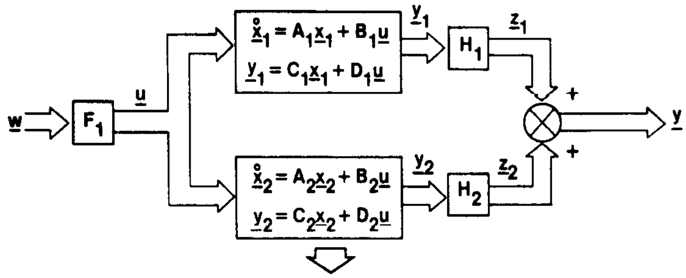
Summation of Systems With a Common Input

Figure 11 illustrates the summation of two systems having a common input. The total system is preceded by an input selection matrix. Output selection matrices are appended to each system before the summation. After rewriting both sets of system equations to include the input vector and input selection matrix, the total system formulation follows immediately from the definition of the total system output vector.

The systems are defined by the standard equations.

$$\left.\begin{aligned}\dot{\underline{x}}_1 &= A_1\underline{x}_1 + B_1\underline{u}\\ \underline{y}_1 &= C_1\underline{x}_1 + D_1\underline{u}\end{aligned}\right\}\ \text{System 1}$$

$$\left.\begin{aligned}\dot{\underline{x}}_2 &= A_2\underline{x}_2 + B_2\underline{u}\\ \underline{y}_2 &= C_2\underline{x}_2 + D_2\underline{u}\end{aligned}\right\}\ \text{System 2}$$



$$\begin{bmatrix} \overset{\circ}{\underline{x}}_1 \\ \hdashline \overset{\circ}{\underline{x}}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \hdashline \underline{x}_2 \end{bmatrix} + \begin{bmatrix} B_1F_1 \\ \hdashline B_2F_1 \end{bmatrix} \underline{w}$$

$$\underline{y} = \begin{bmatrix} H_1C_1 & H_2C_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \hdashline \underline{x}_2 \end{bmatrix} + \begin{bmatrix} H_1D_1F_1 + H_2D_2F_1 \end{bmatrix} \underline{w}$$

Figure 11. State representation of summation of systems with a common input.

The common control vector $\underline{u}$ for two systems is defined in terms of the input vector $\underline{w}$ and the input selection matrix F_1.

$$\underline{u} = F_1\underline{w}$$

The total system output vector can be written in terms of the output vectors of the two systems,

$$\underline{y} = \underline{z}_1 + \underline{z}_2$$

or expanded in terms of the output selection matrices and observation vectors of the two systems.

$$\underline{y} = H_1\underline{y}_1 + H_2\underline{y}_2$$

The state vector $\underline{x}$ and the control vector $\underline{u}$ for the total system are defined as follows.

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \hdashline \underline{x}_2 \end{bmatrix}$$

$$\underline{u} = \underline{w}$$

Rewriting the state equations in terms of the input vector yields

$$\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1F_1\underline{w} \quad \text{System 1}$$

$$\dot{\underline{x}}_2 = A_2\underline{x}_2 + B_2F_1\underline{w} \quad \text{System 2}$$

The observation vector can be written by substituting for the observation vectors in the expanded observation vector definition.

$$\underline{y} = H_1C_1\underline{x}_1 + H_2C_2\underline{x}_2 + (H_1D_1F_1 + H_2D_2F_1)\underline{w}$$

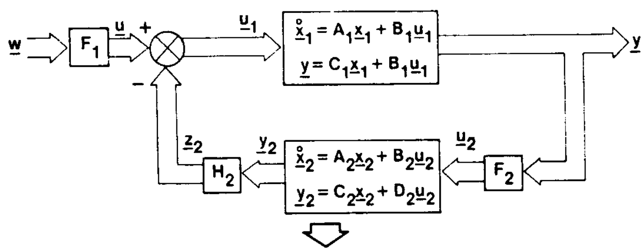
Using these equations, the total system can then be formulated in terms of partitioned matrices.

$$\left[\begin{array}{c} \dot{\underline{x}}_1 \\ \hline \dot{\underline{x}}_2 \end{array}\right] = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_2 \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline \underline{x}_2 \end{array}\right] + \left[\begin{array}{c} B_1F_1 \\ \hline B_2F_1 \end{array}\right] \underline{w}$$

$$\underline{y} = \left[\begin{array}{c|c} H_1C_1 & H_2C_2 \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline \underline{x}_2 \end{array}\right] + \left[H_1D_1F_1 + H_2D_2F_1\right] \underline{w}$$

Feedback System

Figure 12 illustrates a generalized feedback system with a linear subsystem in the forward and feedback loops. An input selection matrix precedes the total system. The observation vector of the forward-loop system is used as the total system output vector. The feedback system incorporates an input and an output selection matrix. After a definition of the total system state, control, and observation vectors, a multistep process is used to rewrite the subsystem equations in terms of the elements of the total system vectors.



$$\left[\begin{array}{c} \overset{\circ}{\underline{x}}_1 \\ \hline \overset{\circ}{\underline{x}}_2 \end{array}\right] \left[\begin{array}{c|c} A_1 - B_1H_2D_2F_2NC_1 & -B_1H_2\left(C_2 - D_2F_2ND_1H_2C_2\right) \\ \hline B_2F_2NC_1 & A_2 - B_2F_2ND_1H_2C_2 \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline \underline{x}_2 \end{array}\right] + \left[\begin{array}{c} B_1F_1 - B_1H_2D_2F_2ND_1F_1 \\ \hline B_2F_2ND_1F_1 \end{array}\right] \underline{w}$$

$$\underline{y} = \left[\begin{array}{c|c} NC_1 & -ND_1H_2C_2 \end{array}\right] \left[\begin{array}{c} \underline{x}_1 \\ \hline x_2 \end{array}\right] + \left[ND_1F_1\right] \underline{w}$$

$$\text{Where } N = \left(I + D_1H_2D_2F_2\right)^{-1}$$

Figure 12. State representation of feedback system.

The equations for the two subsystems are given as

$$\left.\begin{aligned} \dot{\underline{x}}_1 &= A_1\underline{x}_1 + B_1\underline{u}_1 \\ \underline{y}_1 &= C_1\underline{x}_1 + D_1\underline{u}_1 \end{aligned}\right\} \text{ System 1}$$

$$\left.\begin{aligned} \dot{\underline{x}}_2 &= A_2\underline{x}_2 + B_2\underline{u}_2 \\ \underline{y}_2 &= C_2\underline{x}_2 + D_2\underline{u}_2 \end{aligned}\right\} \text{ System 2}$$

The control vector to the first system, $\underline{u}_1$, is defined in terms of the total system input vector $\underline{w}$, the total system selection matrix F_1, and the feedback vector $\underline{z}_2$.

$$\begin{aligned} \underline{u}_1 &= \underline{u} - \underline{z}_2 \\ &= F_1\underline{w} - \underline{z}_2 \end{aligned}$$

This can be expanded in terms of the observation vector of the second system, $\underline{y}_2$, and the output selection matrix of the second system, H_2, using the definition

$$\underline{z}_2 = H_2\underline{y}_2$$

and substituting into the equation for the control vector for the first system, $\underline{u}_1$, yields

$$\underline{u}_1 = F_1\underline{w} - H_2\underline{y}_2$$

The control vector for the second (feedback) system is defined as

$$\underline{u}_2 = F_2\underline{y}_1$$

The state, control, and observation vectors for the total system are defined as follows.

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix}$$

$$\underline{u} = \underline{w}$$

$$\underline{y} = \underline{y}_1$$

The equations for the total system are derived by first expanding the observation equation for the forward-loop system using the expanded equation for the control vector of the first system.

$$\underline{y}_1 = C_1\underline{x}_1 + D_1(F_1\underline{w} - H_2\underline{y}_2)$$

$$= C_1\underline{x}_1 + D_1F_1\underline{w} - D_1H_2\underline{y}_2$$

Substituting the observation equation for the second system into this equation yields

$$\underline{y}_1 = C_1\underline{x}_1 + D_1F_1\underline{w} - D_1H_2(C_2\underline{x}_2 + D_2\underline{u}_2)$$

$$= C_1\underline{x}_1 + D_1F_1\underline{w} - D_1H_2C_2\underline{x}_2 - D_1H_2D_2\underline{u}_2$$

This equation is expanded again in terms of the definition for the control vector for the feedback system

$$\underline{y}_1 = C_1\underline{x}_1 + D_1F_1\underline{w} - D_1H_2C_2\underline{x}_2 - D_1H_2D_2(F_2\underline{y}_1)$$

$$= C_1\underline{x}_1 + D_1F_1\underline{w} - D_1H_2C_2\underline{x}_2 - D_1H_2D_2F_2\underline{y}_1$$

which can be written as

$$(I + D_1H_2D_2F_2)\underline{y}_1 = C_1\underline{x}_1 + D_1F_1\underline{w} - D_1H_2C_2\underline{x}_2$$

By defining an intermediate matrix, N, for notational convenience, as

$$N = (I + D_1H_2D_2F_2)^{-1}$$

the observation equation for the forward-loop system can be written

$$\underline{y}_1 = NC_1\underline{x}_1 - ND_1H_2C_2\underline{x}_2 + ND_1F_1\underline{w}$$

In a similar manner, the observation equation for the feedback system can be derived. Starting with the simple equation for the observation vector of the second system, $\underline{y}_2$,

$$\underline{y}_2 = C_2\underline{x}_2 + D_2\underline{u}_2$$

and substituting for $\underline{u}_2$ yields

$$\underline{y}_2 = C_2\underline{x}_2 + D_2F_2\underline{y}_1$$

This equation can be expanded in terms of $\underline{y}_1$ to give

$$\underline{y}_2 = C_2\underline{x}_2 + D_2F_2\left(NC_1\underline{x}_1 - ND_1H_2C_2\underline{x}_2 + ND_1F_1\underline{w}\right)$$

$$= D_2F_2NC_1\underline{x}_1 + (C_2 - D_2F_2ND_1H_2C_2)\underline{x}_2 + D_2F_2ND_1F_1\underline{w}$$

Using this equation for the observation vector of the feedback system, the state equation for the first system can be written. Using the original state equation

$$\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1\underline{u}_1$$

and the expanded definition of the control vector for the first system,

$$\underline{u}_1 = F_1\underline{w} - H_2\underline{y}_2$$

the state equation becomes

$$\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1(F_1\underline{w} - H_2y_2)$$

$$= A_1\underline{x}_1 + B_1F_1\underline{w} - B_1H_2\underline{y}_2$$

which can be expanded in terms of the observation vector for the second system, $\underline{y}_2$, to give

$$\dot{\underline{x}}_1 = A_1\underline{x}_1 + B_1F_1\underline{w}$$

$$- B_1H_2\left[D_2F_2NC_1\underline{x}_1 + (C_2 - D_2F_2ND_1H_2C_2)\underline{x}_2 + D_2F_2ND_1F_1\underline{w}\right]$$

Rearranging terms yields

$$\dot{\underline{x}}_1 = (A_1 - B_1H_2D_2F_2NC_1)\underline{x}_1 - B_1H_2(C_2 - D_2F_2ND_1H_2C_2)\underline{x}_2$$

$$+ (B_1F_1 - B_1H_2D_2F_2ND_1F_1)\underline{w}$$

Finally, an expression for the state equation for the second system is derived. This derivation is based on the definition of the control vector for the feedback system $\underline{u}_2$,

$$\underline{u}_2 = F_2\underline{y}_1$$

and the final expression for $\underline{y}_1$,

$$\underline{y}_1 = NC_1\underline{x}_1 - ND_1H_2C_2\underline{x}_2 + ND_1F_1\underline{w}$$

which expands the definition to

$$\underline{u}_2 = F_2NC_1\underline{x}_1 - F_2ND_1H_2C_2\underline{x}_2 + F_2ND_1F_1\underline{w}$$

This expression for $\underline{u}_2$ is substituted into the original state equation for the feedback system

$$\dot{\underline{x}}_2 = A_2\underline{x}_2 + B_2\underline{u}_2$$

to yield

$$\dot{\underline{x}}_2 = A_2\underline{x}_2 + B_2\left(F_2NC_1\underline{x}_1 - F_2ND_1H_2C_2\underline{x}_2 + F_2ND_1F_1\underline{w}\right)$$

Rearranging terms results in a usable expression for the state equation of the second system.

$$\dot{\underline{x}}_2 = B_2F_2NC_1\underline{x}_1 + (A_2 - B_2F_2ND_1H_2C_2)\underline{x}_2 + B_2F_2ND_1F_1\underline{w}$$

The final state equations and the observation equation for the first system can be expressed in terms of partitioned matrices for the total system equation.

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \hline \dot{\underline{x}}_2 \end{bmatrix} = \left[\begin{array}{c|c} A_1 - B_1H_2D_2F_2NC_1 & -B_1H_2(C_2 - D_2F_2ND_1H_2C_2) \\ \hline B_2F_2NC_1 & A_2 - B_2F_2ND_1H_2C_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix}$$

$$+ \left[\begin{array}{c} B_1F_1 - B_1H_2D_2F_2ND_1F_1 \\ \hline B_2F_2ND_1F_1 \end{array}\right] \underline{w}$$

$$\underline{y} = \left[\begin{array}{c|c} NC_1 & -ND_1H_2C_2 \end{array}\right] \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix} + \left[ND_1F_1\right] \underline{w}$$

MATRIX$_X$ COMMAND FILE

To aid in the process of combining and connecting linear multi-input, multi-output subsystems into a total system model, a command file has been created in MATRIX$_X$ format. This file uses subsystem matrices whose names are consistent with the notation of this report and produces total systems matrices ANEW, BNEW, CNEW, and DNEW which correspond to the A, B, C, and D matrices, respectively. The options provided by this command file are the six sample systems developed in this report. In addition to the four matrices above, the command file produces a total system matrix, SNEW, defined in the following way.

$$SNEW = \left[\begin{array}{c|c} ANEW & BNEW \\ \hline CNEW & DNEW \end{array}\right]$$

While the command file combines subsystems two at a time, the file is designed to allow multiple combinations without leaving the command file.

The command file is used within $MATRIX_X$ by typing

EXEC('COMBLK')

The command file responds by typing a title,

COMMAND FILE TO ASSIST IN COMBINING AND CONNECTING SYSTEMS

REFERENCE NASA TM-85912

After the title is printed, the screen is cleared and a menu is displayed.

WANT TO DEFINE MATRICES

(1) NO
(2) YES

The selection is made by positioning the cursor next to the desired answer and then depressing the RETURN key.

If all matrices have been named using the notation of this report, the matrices do not have to be redefined. However, if other names have been used, the command file will request that the user define the appropriate matrices. A typical response following an answer of "yes" to the question "want to define blocks" is

ENTER THE STATE MATRIX FOR THE FIRST SYSTEM, A1

The user must then either enter the elements of the A_1 matrix or equate the A_1 matrix to another, previously entered matrix, representing the state matrix for the first system.

After the matrices for both systems are defined in the correct notation, the screen is again cleared, and another menu appears.

SELECT AN OPTION

(1) PARALLEL SYSTEMS
(2) SUMMATION
(3) CONCATENATION
(4) COMMON INPUT SYSTEMS
(5) SUM COMMON INPUT SYST
(6) FEEDBACK SYSTEM

Again, the selection is made by positioning the cursor next to the desired option and then depressing the RETURN key. The options listed correspond to the examples detailed in this report.

If the user has elected to enter or define the matrices, the command file will again request that the required input and output selection matrices be entered. The actual matrices to be defined are determined by the option selected. However, because all options require an input selection matrix for the first system, F_1, the following message will always appear if the user is entering or defining matrices.

ENTER THE INPUT SELECTION MATRIX FOR THE FIRST SYSTEM, F1

The user must then either enter the elements of the F_1 matrix or equate the F_1 matrix to another, already entered matrix, representing the input selection matrix for the first system.

The screen will again be cleared and another menu will appear.

DO YOU WANT TO EXIT

(1) NO
(2) YES

The selection is made by positioning the cursor next to the desired answer and depressing the RETURN key. If (1) is selected, the command file will return to the first menu and again ask if the user desires to define the matrices. Selection of (2) will cause the system to exit the command file and return the user to MATRIX_X.

Ames Research Center
Dryden Flight Research Facility
National Aeronautics and Space Administration
Edwards, Calif., August 2, 1984

REFERENCE

1. MATRIX_X User's Guide. Integrated Systems Inc., 151 University Ave., Palo Alto, Calif., 1982.

APPENDIX A — LISTING OF COMMAND FILES

There are eleven command files to assist in combining and connecting systems. These command files each perform specific functions described below. The proliferation of these files is partly due to normal program structuring and partly the result of a 1024-character limitation on command files within MATRIX_X.

The COMBLK command file (fig. A.1) is the main file used to combine and connect systems. This file acts as an executive and controls the basic operation of the command file. MENUS (fig. A.2) is a command file to define the menus listed when using the COMBLK command file. The INPSYS command file (fig. A.3) is used to provide the user prompting while entering the system matrices. The input of selection matrices is controlled by the SELECT command file (fig. A.4). SELECT contains logic to request only the selection matrices required for the application selected by the user. The actual combining and connecting of systems is controlled by the EXCOPT command file (fig. A.5).

Figures A.6 to A.11 list the command files used to determine the matrices for the total combined system. The combining of parallel systems into a single system description is done by the PARSYS command file (fig. A.6). The summation of parallel systems is performed by the SUMSYS command file (fig. A.7). The CONSYS command file (fig. A.8) determines the total system matrices when the systems are to be concatenated. The COMINP (fig. A.9) and SUMCOM (fig. A.10) command files determine the total system matrices for systems with a common input and separate outputs or with a common input and a summed output. The FEDBAK command file (fig. A.11) determines the total system matrices for a feedback system.

```
//

//      COMMAND FILE TO ASSIST IN COMBINING SYSTEMS

//

//      REFERENCE NASA TM-85912

//      LEE DUKE

//

EXEC('MENUS')

DISP('COMMAND FILE TO ASSIST IN COMBINING AND CONNECTING SYSTEMS')

DISP('REFERENCE NASA TM-85912')

ANSWER3=1

WHILE ANSWR3=1; . . .

       ANSWR1=MENU(LISTOP)-1; . . .

       IF ANSWR1=1;EXEC('INPSYS');END; . . .

       ANSWR2=MENU(LISTCM); . . .

       IF ANSWR1=1;EXEC('SELECT');END; . . .

       EXEC('EXCOPT'); . . .

       ANSWR3=MENU(LISTCH); . . .

END;
```

Figure A.1 Listing of COMBLK command file.

```
//
//      COMMAND FILE TO SET UP MENUS FOR COMBINING SYSTEMS
//
LISTOP = ['WANT TO DEFINE BLOCKS'
          'NO                   '
          'YES                  ']

LISTCM = ['SELECT AN OPTION     '
          'PARALLEL SYSTEMS     '
          'SUMMATION            '
          'CONCATENATION        '
          'COMMON INPUT SYSTEMS '
          'SUM COMMON INPUT SYST'
          'FEEDBACK SYSTEM      '];

LISTCH = ['DO YOU WANT TO EXIT'
          'NO                   '
          'YES                  '];
```

Figure A.2 Listing of MENUS command file.

```
//
//         COMMAND FILE TO INPUT SYSTEM MATRICES
//
DISP ('ENTER THE STATE MATRIX FOR THE FIRST SYSTEM, A1');
EXEC(5);
DISP ('ENTER THE CONTROL MATRIX FOR THE FIRST SYSTEM, B1');
EXEC(5);
DISP ('ENTER THE OBSERVATION MATRIX FOR THE FIRST SYSTEM, C1')
EXEC(5);
DISP ('ENTER THE FEEDFORWARD MATRIX FOR THE FIRST SYSTEM, D1');
EXEC(5);
DISP ('ENTER THE STATE MATRIX FOR THE SECOND SYSTEM, A2');
EXEC(5);
DISP ('ENTER THE CONTROL MATRIX FOR THE SECOND SYSTEM, B2');
EXEC(5);
DISP ('ENTER THE OBSERVATION MATRIX FOR THE SECOND SYSTEM, C2');
EXEC(5);
DISP ('ENTER THE FEEDFORWARD MATRIX FOR THE SECOND SYSTEM, D2');
EXEC(5);
```

Figure A.3 Listing of INPSYS command file.

```
//

//        COMMAND FILE TO INPUT SELECTION MATRICES

//

DISP ('ENTER THE INPUT SELECTION MATRIX FOR THE FIRST');

DISP ('SYSTEM, F1');

EXEC(5);

IF ANSWR2 < > 3; IF ANSWR2 < > 4; IF ANSWR2 < > 5; . . .

     DISP ('ENTER THE INPUT SELECTION MATRIX FOR THE SECOND'); . . .

     DISP ('SYSTEM, F2'); . . .

     EXEC(5); . . .

END;END;END;

IF ANSWR2 < > 1; IF ANSWR2 < > 4; IF ANSWR2 < > 5; . . .

     DISP ('ENTER THE SELECTION MATRIX FOR THE OUTPUT OF THE FIRST');. . .

     DISP ('SYSTEM, H1'); . . .

     EXEC(5); . . .

END;END;END;

IF ANSWR2 < > 1; IF ANSWR2 > 4; . . .

     DISP ('ENTER THE SELECTION MATRIX FOR THE OUTPUT OF THE SECOND');. . .

     DISP ('SYSTEM, H2'); . . .

     EXEC(5); . . .

END;END;
```

Figure A.4 Listing of SELECT command file.

```
//
//   COMMAND FILE TO EXECUTE OPTIONS FOR COMBINING
//   AND CONNECTING SYSTEMS
//
IF ANSWR2 = 1; EXEC ('PARSYS'); END;
IF ANSWR2 = 2; EXEC ('SUMSYS'); END;
IF ANSWR2 = 3; EXEC ('CONSYS'); END;
IF ANSWR2 = 4; EXEC ('COMINP'); END;
IF ANSWR2 = 5; EXEC ('SUMCOM'); END;
IF ANSWR2 = 6; EXEC ('FEDBAK'); END;
SNEW = [ANEW BNEW; CNEW DNEW];
```

Figure A.5 Listing of EXCOPT command file.

```
//
//   COMMAND FILE TO COMBINE PARALLEL SYSTEMS
//
<N1,M1> = SIZE(B1); <R1,M1> = SIZE(D1); <M1,L1> = SIZE(F1);
<N2,M2> = SIZE(B2); <R2,M2> = SIZE(D2); <M2,L2> = SIZE(F2);
ANEW = [A1 0*ONES(N1,N2);0*ONES(N2,N1) A2];
BNEW = [B1*F1 0*ONES(N1,L2); 0*ONES(N2,L1) B2*F2];
CNEW = [C1 0*ONES(R1,N2); 0*ONES(R2,N1) C2];
DNEW = [D1*F1 0*ONES(R1,L2); 0*ONES(R1,L1) D2*F2];
```

Figure A.6 Listing of PARSYS command file.

```
//
//   COMMAND FILE TO SUM PARALLEL SYSTEMS
//
<N1,M1> = SIZE(B1); <M1,L1> = SIZE(F1);
<N2,M2> = SIZE(B2); <M2,L2> = SIZE(F2);
ANEW = [A1 0*ONES(N1,N2); 0*ONES(N2,N1) A2];
BNEW = [B1*F1 0*ONES(N2,L2); 0*ONES(N2,L1) B2*F2];
CNEW = [H1*C1 H2*C2];
DNEW = [H1*D1*F1  H2*D2*F2];
```

Figure A.7 Listing of SUMSYS command file.

```
//
//   COMMAND FILE TO CONCATENATE SYSTEMS


//
<N1,M1> = SIZE(B1);
<N2,M2> = SIZE(B2);
ANEW = [A2 0*ONES(N1,N2); B2*H1*C1 A2];
BNEW = [B1*F1; B2*H1*D1*F1];
CNEW = [D2*H1*C1  C2];
DNEW = [D2*H1*D1*F1];
```

Figure A.8 Listing of CONSYS command file.

```
//
//   COMMAND FILE TO COMBINE FEEDBACK SYSTEM
//
<N1,M1> = SIZE(B1);
N = INV(EYE(N1) + D1*H2*D2*F2);
ANEW = [A1-B1*H2*D2*F2*N*C1  -B1*H2*(C2-D2*F2*N*D1*H2*C2).
        B2*F2*N*C1              A2-B2*F2*N*D1*H2*C2];
BNEW = [B1*F1-B2*H2*D2*F2*N*D1*F1; B2*F2*N*D1*F1];
CNEW = [N*C1 -N*D1*H2*C2];
DNEW = [N*D1*F1];
```

Figure A.11 Listing of FEDBAK command file.

1. Report No. NASA TM-85912	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Combining and Connecting Linear, Multi-Input, Multi-Output Subsystem Models		5. Report Date April 1986
		6. Performing Organization Code
7. Author(s) Eugene L. Duke		8. Performing Organization Report No. H-1264
9. Performing Organization Name and Address NASA Ames Research Center Dryden Flight Research Facility P.O. Box 273 Edwards, CA 93523-5000		10. Work Unit No. RTOP 505-34-01
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Memorandum
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

The mathematical background for combining and connecting linear, multi-input, multi-output subsystem models into an overall system model is presented. Several sample subsystem configurations are examined in detail, and these sample subsystems and the resulting system models are illustrated. A $MATRIX_X$ command file to aid in combining and connecting systems is also presented and its use is described.

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